

Critical Behavior of Two Interacting Linear Polymer Chains in a Good Solvent

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A model of two interacting (chemically different) linear polymer chains is solved exactly using the real-space renormalization group transformation on a family of Sierpinski gasket type fractals and on a truncated 4-simplex lattice. The members of the family of the Sierpinski gasket-type fractals are characterized by an integer scale factor b which runs from 2 to ∞ . The Hausdorff dimension d_F of these fractals tends to 2 from below as $b \rightarrow \infty$. We calculate the contact exponent γ for the transition from the state of segregation to a state in which the two chains are entangled for $b = 2-5$. Using arguments based on the finite-size scaling theory, we show that for $b \rightarrow \infty$, $\gamma = 2 - \nu(b) d_F$, where ν is the end-to-end distance exponent of a chain. For a truncated 4-simplex lattice it is shown that the system of two chains either remains in a state in which these chains are intermingled in such a way that they cannot be told apart, in the sense that the chemical difference between the polymer chains completely drop out of the thermodynamics of the system, or in a state in which they are either zipped or entangled. We show the region of existence of these different phases separated by tricritical lines. The value of the contact exponent γ is calculated at the tricritical points.

KEY WORDS: Segregation; entanglement; tricritical line; contact exponent; finite-size scaling; fractals.

1. INTRODUCTION

In order to explain the entanglement and segregation of polymer chains in a solution a lattice model of two interacting self-avoiding-walks (SAWs) has recently been proposed.⁽¹⁾ In this model walks were allowed to cross each other at most once on a lattice point and a lattice bond was allowed to be occupied at most by a step of one or by both walks. This model hereinafter referred to as a model of two interacting crossed walks or

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TICWs, represents a physical system of two chemically different interacting charge neutral homo polymer chains in a solution in which like monomers repel each other while unlike monomers interact with a relatively short-range attraction. The crossing of walks at a lattice point corresponds to the situation in which the length which characterizes the attractions between unlike monomers is less than that of the repulsion between like monomers. Such an attraction may arise due to formation of chemical bonds between unlike monomers or due to solvent mediated interactions.

The model of TICWs was solved exactly for a truncated 3-simplex lattice using a real space renormalization group (RSRG) transformation.⁽¹⁾ The phase diagram which emerged from this calculation has a tricritical point for a transition from a state of segregation in which chains are separated from each other without any overlap to a state in which the two chain get entangled. In the vicinity of this point a cross-over regime is observed. The mean number of monomers M of one chain in contact with other chain at the transition point was found to behave as

$$M \propto N^y \quad (1.1)$$

where N is the total number of monomers in a chain and y is the contact exponent.

One of the motivations of this paper is to solve the model of TICWs and calculate the value of y for the members of a family of Sierpinski gasket type fractals embedded in a 2-dimensional space. The members of this family are characterized by a scale parameter b which runs from 2 to ∞ . The fractal dimension d_F of a member of the family is written in terms of b as⁽²⁾

$$d_F = \frac{\log[b(\frac{b+1}{2})]}{\log b} \quad (1.2)$$

For the spectral dimension \tilde{d} we, however, do not have such a simple expression. For large b one has the following asymptotic series⁽³⁾

$$\tilde{d}(b) = 2 - \frac{\log \log b}{\log b} + \text{terms of order } \left(\frac{1}{\log b} \right) \quad (1.3)$$

One therefore sees from Eqs. (1.2) and (1.3) that as b is increased the fractal and spectral dimensions increase monotonically and tend towards 2 from below and for $b \rightarrow \infty$ the lattice resembles a 2- d plane.

The bulk critical exponents of a SAW have been evaluated for the members of this family of fractals using the RSRG method by Elezovic *et al.*⁽⁴⁾ for $2 \leq b \leq 8$ and by the Monte Carlo method for $2 \leq b \leq 100$.⁽⁵⁾ For

large b the finite size scaling arguments have been used⁽⁶⁾ to calculate the value of the exponents ν (associated with the end to end distance) and γ (associated with the total number of distinct SAWs). It is shown that while ν approaches to its $2-d$ value from below as $b \rightarrow \infty$, γ is about three times (133/43) larger than the corresponding Euclidean value of 43/32. It is therefore interesting to examine the behavior of γ as a function of b for this family of lattices.

The other motivation stems from our earlier study⁽⁷⁾ of a model of two interacting self-attracting-self-avoiding walks (SASAWs) with the constraints that these walks are not allowed to cross each other at any lattice point and a lattice bond can be occupied at most once by a step of any one walk. This model has been referred to as a model of two interacting walks or TIWs. This model has been solved exactly using the RSRG transformations for a family of truncated n -simplex lattices with $4 \leq n \leq 6$. Here, we solve the model of TICWs for a truncated 4-simplex lattice and compare our results with the results found for TIWs in ref. 7 and show that the latter is a special class of the former.

The paper is organized as follows. In Section 2 we describe the real space renormalization group (RSRG) method to calculate the contact exponents for the model of TICWs for a fractal lattice. We report results for the members of a family of Sierpinski gasket-type fractal lattice. In Section 3 we consider a truncated 4-simplex lattice and solve the model of TICWs and compare the result with that of a model of TIWs considered in a previous paper.⁽⁷⁾ The paper ends with a brief discussion given in Section 4.

2. REAL SPACE RENORMALIZATION GROUP CALCULATION OF CONTACT EXPONENT γ FOR A FAMILY OF SIERPINSKI GASKET-TYPE FRACTALS

We consider the model of TICWs on a family of Sierpinski gasket-type fractals and associate contact energy E_C with each crossing of walks. For notational simplification we denote these walks hereinafter as P_1 and P_2 . The generating function of our interest is

$$G(x_1, x_2, \omega, t) = \sum_{\text{all walks}} x_1^{N_1} x_2^{N_2} \omega^{R_1} t^{R_2} \quad (2.1)$$

where $N_1(N_2)$ is the number of steps (monomers) in a walk (polymer) $P_1(P_2)$ and $x_1(x_2)$ denotes the fugacity weight attached to each step of walk $P_1(P_2)$. R_1 is the total number of sites visited by both the walks and R_2 is number of pairs of sites which are adjacent to the doubly visited sites

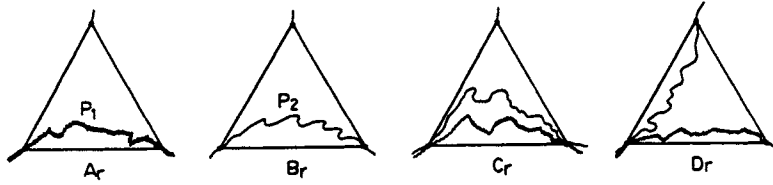


Fig. 1. Diagrams representing the four restricted generating functions for the chains (indicated by smooth (P_1) and wiggle (P_2) lines) on a Sierpinski gasket.

and each site of the pair is visited by a different walk. $\omega = \exp(E_C/T)$ and $t = \exp(E_t/T)$ where E_C , as mentioned above, is the contact energy and E_t is the energy associated with an unoccupied lattice bond adjacent to the doubly visited site.

From the generating function the average length of a polymer chain (say P_1) is calculated using the relation

$$\langle N_1 \rangle = x_1 \frac{\partial \ln G}{\partial x_1} \quad (2.2)$$

Similarly the number of monomers in contact with each other (or the mean number of sites visited by both walks) can be calculated from the relation

$$\langle M \rangle = \omega \frac{\partial \ln G}{\partial \omega} \quad (2.3)$$

Calculation of G for any member of the lattice considered here requires four restricted partition functions shown in Fig. 1. These partition functions are defined recursively as a weighted sum over all configurations for a given stage of the iterative construction of the fractal lattice. The recursions express the restricted functions for the $(r+1)$ th-order lattice in terms of those of the r th generation one. The variables in these equations are just the partial generating functions corresponding to different polymer configurations for a given size of the fractal lattice.

To illustrate the procedure of calculating the contact exponent using the RSRG approach and to set the notations we consider a lattice with scale parameter $b=3$. The recursion relations for this lattice are easily found and can be written as

$$A_{r+1} = A^3 + 3A^4 + A^5 + 2A^6 \quad (2.4)$$

$$B_{r+1} = B^3 + 3B^4 + B^5 + 2B^6 \quad (2.5)$$

$$\begin{aligned}
C_{r+1} = & C^3 + 3C^4 + C^5 + 2C^6 + 4ABC^2D + 2CD^5 \\
& + 2CD^2(A + A^2B + A^3B + AB^2 + B^3) \\
& + D^2(A^2B + A^3B + AB^2 + AB^3 + AC^2 \\
& + BC^2 + 2AC^3 + 2BC^3) + 2D^3(AB + A^2C + B^2C) \\
& + 2(A^2 + B^2)(D^4 + C^3D)
\end{aligned} \tag{2.6}$$

$$\begin{aligned}
D_{r+1} = & D(A^2(B^2 + AB^2 + 2BC + C^3 + B^3) \\
& + B^2(2AC + C^3)) + 2D^2(2A^2B^2 + A^2C^2 + B^2C^2) \\
& + D^3(A(A^2 + B + 2B^2 + C^2) + B(2A^2 + B^2 + C^2)) \\
& + D^4(2A^2 + 2B^2 + C^2) + D^5(A + B + 2C + D) \\
& + CD^3(A + A^2 + B + B^2 + AB) + 2A^2B^2C^2 \\
& + C^2D(A^2 + A^3 + B^2 + B^3 + 2AB)
\end{aligned} \tag{2.7}$$

Here and below we adopt a notational simplification in which the index r is dropped from the right hand side of the recursion relations.

It may be emphasized here that the recursion relations written above are exact for the model of TICWs. The fact that recursion relation for $A(B)$ is independent of $B(A)$, C and D is the consequence of the definition of the model. This means that the model of TICWs as defined above is unable to take into account the effect of one SAW on the self-avoidance of the other SAW. This will, however, not affect the essential features of the phase diagram to be calculated below. This is because in the dilute solution the criticality of a chain does not get affected due to the presence of other chain.

The recursion relation of $A(B)$ which is the weight of an $r + 1$ th order triangle in which chain $P_1(P_2)$ enters in from one of the corner vertices and comes out from the other, is independent of $B(A)$, C and D . The effects of interactions between two chains are taken through C and D as illustrated in Fig. 2. Since the interaction between two chains are restricted to bonds within a first order unit of the fractal lattice, ω and t do not appear explicitly in the recursion equations. They appear only in the initial value of the generating functions given below. By universality one, however, expects that the qualitative phase diagrams and critical exponent of this system are same as the one with interactions which are of much longer length scales but smaller than the correlation length which diverges at the critical point.

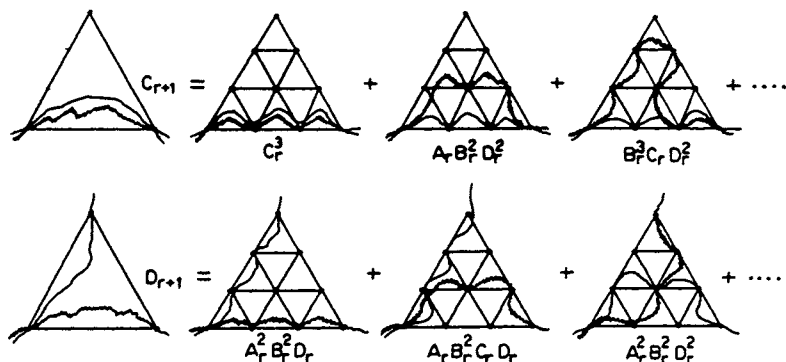


Fig. 2. Some of the diagrams (out of 98) representing the recursion relations for the two functions (C and D) given by Eqs. (2.6) and (2.7) for a Sierpinski gasket-type fractal with $b=3$.

In Fig. 2 we illustrate the configuration of order r which appear in Eqs. (2.4)–(2.7). The starting weight for these functions are

$$\begin{aligned} A_0 &= x_1 \\ B_0 &= x_2 \\ C_0 &= x_1 x_2 \omega^2 \end{aligned} \quad (2.8)$$

and

$$D_0 = x_1 x_2 t \omega$$

In the asymptotic limit both N_1 and N_2 tend to infinity, and therefore, in that situation $x_1 = x_2$ and $A_{r+1} = B_{r+1}$. From these equations we get the following three nontrivial fixed points.

(i) The fixed point $(A^* = B^*, C^*, D^*) = (0.5511, 0, 0)$ corresponds to the bulk segregated state. Linearization of the recursion relations around this fixed point gives one eigenvalue greater than one, $\lambda_1 = 3.9919$ which gives the radius of gyration exponent $\nu = 0.7936^{(8)}$ of a chain. For $x = x_C(\omega) = 0.5511$ this fixed point is reached for all values of $\omega < \omega_C(t)$.

(ii) The fixed point $(A^* = B^*, C^*, D^*) = (0, 0.5511, 0)$ is reached for all $\omega > \omega_C(t)$ and $x < x_C(\omega)$. This fixed point corresponds to a state in which both chains are entangled with each other. In some approximate sense this state is reminiscent of a “double helix” kind of structure of a DNA molecule. This is because of the fact that in this state both chains are

coupled together (see Fig. 1) and behave like flexible ladder which is similar to the double stranded helix kind structure of DNA molecule. The local structure of the entangled polymer chains may, however, be very different from the helical structure of a DNA molecule. Therefore, the similarity drawn here between the DNA double helix kind of structure and that of entangled polymer chains are only at length scales much larger than the helix pitch of the DNA molecule.

Linearization around this fixed point yields two eigenvalues ($\lambda_1 = 3.9919$) with $\nu = 0.7936$. Hence, in this state the two chains entangled with each other in such a way that they behave as one composite chain having the same radius of gyration exponent as that of one chain.

(iii) At $\omega = \omega_c(t)$, $x_c(\omega)$ is still equal to its bulk value. The fixed point which is reached with this value of ω is $(A^* = B^*, C^*, D^*) = (0.5511, 0.3037, 0.3037)$. Linearization of the recursion relations about this fixed point leads two eigenvalues greater than one, $\lambda_1 = 3.9919$ and $\lambda_2 = 2.7267$. We identify this fixed point as a tricritical point of transition from a state of segregation to a state of entanglement. The value of the contact exponent is

$$y = \frac{\ln \lambda_2}{\ln \lambda_1} = 0.7246$$

We now extend this method to other lattices of this family. As the value of b increases, however, the number of possible configurations of different walks increases rapidly. Therefore, the extension of this method to much larger values of b appears difficult as the computer time needed to generate the exact recursion relations by direct enumeration increases as $\exp(b^2)$. We summarize results for $2 \leq b \leq 5$ in Table I in which we list the fixed points corresponding to the tricritical points of the system, the values of two eigenvalues which are greater than one and values of the exponents ν , α and y . The values of ν and α given in the table are in agreement with those reported earlier.^(4, 5, 8)

Table I

b	d_f	$A^* = B^*$	$C^* = D^*$	λ_b	λ_c	ν	y Exact	$\alpha = 2 - d_f \nu$
2	1.5849	0.6180	0.3819	2.3819	1.9159	0.7986	0.7491	0.7342
3	1.6309	0.5511	0.3037	3.9919	2.7267	0.7936	0.7246	0.7056
4	1.6609	0.5063	0.2563	5.8029	3.4957	0.7884	0.7117	0.6905
5	1.6826	0.4745	0.2251	7.7898	4.2443	0.7840	0.7042	0.6808

It may be of interest to note that at the fixed points corresponding to the tricritical points of the systems, the value of $C^* = D^*$ is just square roots of $A^* = B^*$ for all values of b listed in Table 1. This is because of the fact that at the tricritical points the effects of attractive and repulsive interactions mutually cancel each other and the number of configurations in C and D are just the square of the number of configurations of A and B , respectively, for any value of b .

We also find that the values of y decrease monotonically as b is increased. This behavior can be understood from the fact that as the dimensionality of the space increases the walks have less chance to cross each other.

We can estimate the value of y for $b \rightarrow \infty$ using following arguments which are based on the finite-size scaling theory.

Since in the model of TICWs the effect of one SAW on the self-avoidance of the other SAW is neglected and the walks are allowed to cross each other, the distribution of monomers remains isotropic as in the case of a single chain. Using the finite-size scaling theory, it has been shown that for self-avoiding-walks at large b , there are approximately $b^{1/\nu}(\ln b)^{2\nu-1/2\nu}$ number of steps.⁽⁶⁾ Since these are "dense SAWs" as their scaled length tends to infinity, they fill the space with nearly uniform densities.^(6,12) Therefore the number of contacts with the chain P_2 is given as average density \times number of steps in P_1

$$\simeq \frac{K_2 b^{1/\nu} (\ln b)^{2\nu-1/2\nu}}{b^{d_F}} \cdot b^{1/\nu} (\ln b)^{2\nu-1/2\nu} \quad (2.9)$$

where K_2 is a constant. This leads to the following simple expression for $y(b)$

$$y(b) = 2 - d_F \nu \left[1 - \frac{2\nu-1}{2} \frac{\ln \ln b}{\ln b} + \text{terms of order } 1/\ln b \right] \\ \simeq 2 - d_F \nu(b) \quad (2.10)$$

where from⁽⁶⁾

$$\frac{1}{\nu(b)} = \frac{1}{\nu} + \frac{2\nu-1}{2\nu} \frac{\ln \ln b}{\ln b} + \text{terms of order } \frac{1}{\ln b}$$

From above equations it is evident that as $b \rightarrow \infty$

$$y \rightarrow 2 - \nu d_F = 1/2 \quad (2.11)$$

where $\nu = 3/4$ and $d_F = 2$. The above Eq. (2.11) can be derived assuming a very simple argument as follows. Let a polymer of chain length 1 interacts with the other chain of same length. The average number of contacts.

$$M \sim \frac{1^2}{b^{d_F}} \sim b^{2/\nu - d_F}$$

Where $1 = b^{1/\nu}$ is taken. Since $M \sim b^{y/\nu}$ we have

$$y = 2 - \nu d_F$$

However to see how y depends on b , one needs Eq. (2.10). Since $2\nu - 1/2$ is positive for $\nu = 3/4$, $y(b)$ approaches the $1/2$ value monotonically as b is increased.

This analysis shows that the exponent y is equal to α for $b = \infty$. As we know, α is associated with the number of closed loops formed by a SAW in the asymptotic limit. At the tricritical point the two chains therefore intermingle in such a way that the number of crossing of chains scale in a same way as that the loops formed by a chain. At smaller values of b , however, the value of y is somewhat larger than that of α .

3. RESULTS FOR THE MODEL OF TICWS FOR A TRUNCATED 4-SIMPLEX LATTICE

The truncated 4-simplex lattice belongs to the universality class of a 3-dimensional Sierpinski gasket and has widely been used to study configurational properties of both the linear and branched polymers in different environmental conditions. The models leading to coil-globule transition,⁽⁹⁾ surface adsorption and collapse transition,⁽¹⁰⁾ interpenetration and zipped states of two interacting chains,⁽⁷⁾ etc., have been solved exactly.

The basic geometrical unit of construction of a 4-simplex lattice is a tetrahedron with 4-corner vertices and bonds between every pair of vertices. Each vertex connected through a direct bond is termed as a nearest neighbor. The value of d_F , \tilde{d} and the connectivity constant (for a SAW) μ are 2.0, 1.5474..., 2.2866..., respectively.

As is shown in Fig. 3 we need 13 restricted partition functions for complete descriptions of the generating functions of our interest. Since here we confine ourselves to a regime where both chains are in a swollen state we neglect terms corresponding to configurations of P , Q , F , F' , G , G' , H and I (see Fig. 4). We keep a term E while similar looking term P and Q have been dropped. This is justified on the ground that at a fixed point which represents a swollen state of both chains, the value corresponding to these

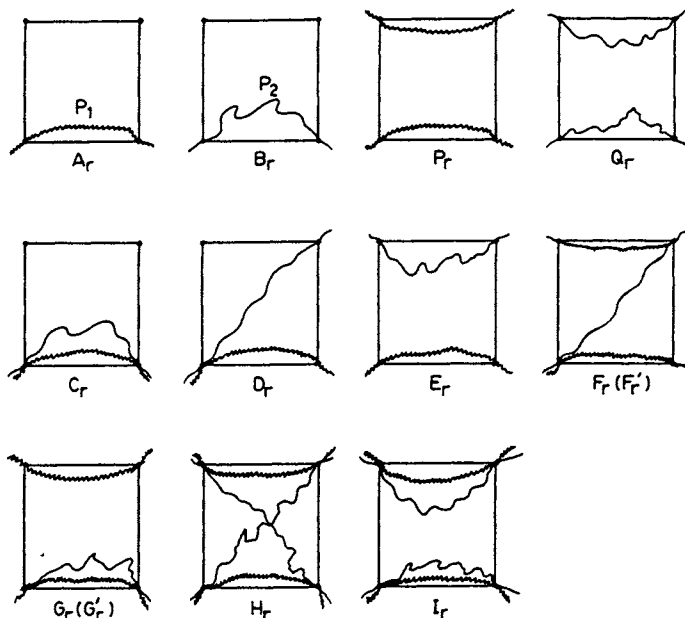


Fig. 3. Diagrams representing the thirteen restricted partition functions for two chains on a 4-simplex lattice, F' and G' have configurations in which chains are interchanged from that of F and G respectively.

configurations are nearly zero. Therefore, their contributions to the eigenvalues of our interest are negligible. To be more specific, let us consider the case of one walk (say P_1) on the 4-simplex lattice. The fixed point corresponding to the swollen state of the walk is $(A^*, P^*) = (0.4294\dots, 0.04998\dots)$. The linearization of the recursion relations about this fixed point leads to an eigenvalue (λ_b) equal to 2.7965 and the exponent $\nu = 0.674$.⁽¹¹⁾ But when we put $P=0$ in the recursion relations the above values get changed to $(A^*, P^*) = (0.4406, 0)$, $\lambda_b = 2.7504$ and $\nu = 0.690$. Thus the error in the value of ν arising due to neglect of terms involving P amount to 2.3%.⁽¹¹⁾ We therefore expect error of this order in the value of γ calculated below.

We are now left with 5 restricted partition functions; the configuration $A(B)$ corresponds to the swollen state of chain $P_1(P_2)$ and other configurations C , D and E correspond to inter-chain correlations. At first look one may wonder as to why P and Q are neglected whereas E is retained. However, on a second reflection one may easily realise that while $P(Q)$ corresponds to intra-chain correlations and is responsible for the collapsed phase of chain $P_1(P_2)$, E corresponds to interchain correlations and is responsible for entangling the chains.

The recursion relations which we consider are

$$A_{r+1} = A^2 + 2A^3 + 2A^4 \quad (3.1)$$

$$B_{r+1} = B^2 + 2B^3 + 2B^4 \quad (3.2)$$

$$C_{r+1} = C^2 + 2C^3 + 2C^4 + 2D^4 + 2D^2[(A+B) + (A^2 + B^2) + 2C(A+B) + AB] \quad (3.3)$$

$$D_{r+1} = ABD(1 + 2E + C) + (A+B)(ABD + CD + DE^2 + 2D^3 + C^2D) + (A^2 + B^2)(CD + DE) + 2(C+E)D^3 + D^3 \quad (3.4)$$

$$E_{r+1} = A^2B^2 + 2ABE(A+B) + 2D^2(A^2 + B^2) + 4D^2E(A+B) + 4ABD^2 + 2D^4 + 2E^4 \quad (3.5)$$

The initial weight given to these functions are

$$\begin{aligned} A_0 &= x_1 \\ B_0 &= x_2 \\ C_0 &= x_1 x_2 \omega^2 \\ D_0 &= x_1 x_2 t \omega \\ E_0 &= x_1 x_2 u^2 \end{aligned} \quad (3.6)$$

Here u represents the nearest neighbor interaction between unlike monomers. In a swollen state of a chain the effect of intra-chain nearest neighbor attraction (in the lattice) is only to change the value of the fugacity and therefore is not considered.

We distinguish between the two situations: A situation in which chains are not allowed to cross, i.e. $\omega = 0$ and the other in which $\omega > 0$.

(A) The case $\omega = 0$ corresponds to the situation where walks repel each other at short distance also and do not allow them to cross each other. Therefore this situation is analogous to the one studied in ref. 7. The critical behaviors of the system are described by the following fixed points.

(i) The fixed point $(A^* = B^*, C^* = D^*, E^*) = (0.4406, \dots, 0, 0.0573\dots)$ is reached for all values of $u < u_c(x_c) = 1.8577\dots$ (see Fig. 4). In ref. 7 we referred this state to as a state of interpenetration. Here the two walks are intermingled with each other in such a way that they cannot be told apart i.e. a walk cannot distinguish between itself and the other walk. This means that the monomers of chains are distributed uniformly without any distinction as if they belong to one chain. The system does not distinguish

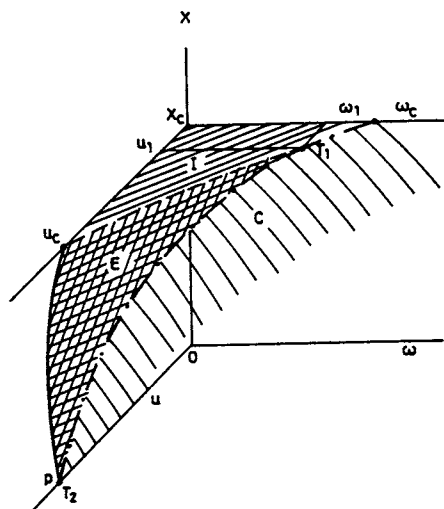


Fig. 4. The complete x - ω - u phase diagram at $t=0.5$ for the truncated 4-simplex lattice. Region marked by I , E and C represent, respectively, the intermingled, zipped and interpenetrated (roped) states of two interacting chains. The tricritical line $u_c\omega_c$ separates the intermingled and zipped states of the system. The dashed-dotted line T_1T_2 separates regions of zipped and interpenetrated (roped) states of the system. The line T_1T_2 appears to meet $u_c\omega_c$ line at T_1 where $\omega = \omega_1 = 1$ and $u = u_1 = 1.8147$. It also appears to meet the line u_cp at p on $\omega = 0$ surface.

between intrapolymer and interpolymer interactions. In this regime therefore, the chemical difference between polymer chains P_1 and P_2 completely drops out of the thermodynamics of the solution as they follow the swollen state statistics and their critical exponents remain the same. Hereinafter we denote this state of two polymer chains by I (see Fig. 4).

(ii) The fixed point ($A^* = B^*$, $C^* = D^*$, $E^* = (0.4406\dots, 0, 0.6700\dots)$) is reached when $u = u_c(x_c)$. Linearization of Eqs. (3.1)–(3.5) about this point gives two distinct eigenvalues greater than one; $\lambda_b = 2.7304\dots$ which corresponds to a swollen state of a walk and $\lambda_c = 2.7484\dots$ which corresponds to the zipped state of the two walks. Using these eigenvalues the values of the contact exponent y is found from the relation

$$y = \frac{\ln \lambda_c}{\ln \lambda_b} \simeq 1$$

This value of y is somewhat higher than the exact value 0.9447 as reported in ref. 7. Here we find an error of about 6%. This, as explained above, is due to neglect of certain restricted partition functions from the recursion relations.

(iii) The fixed point $(A^* = B^*, C^* = D^*, E^*) = (0, 0, 0.7967)$ is reached when $x < x_C$ and $u > u_C(\omega = 0)$. The value of x and u for which this fixed point is reached is shown by line $u_C p$ in Fig. 4. The line remains in the $\omega = 0$ plane. The fixed point, however, corresponds to a state when the two chains exist only in a zipped form and not separately. Since $x < x_C$, the individual chains have not attained their criticality.

(B) For the case $\omega > 0$ the critical behavior of the system is described by the following fixed points.

(iv) When $u < u_C (= 1.8577)$, $x = x_C (= 0.4406)$ and $\omega < \omega_C(u)$ system evolves to a fixed point $(A^* = B^*, C^* = D^*, E^*) = (0.4406, 0, 0.0573)$ corresponding to the state I discussed in (i) above. The plane I in Fig. 4 represents the value of u and ω at $x = x_C$ for which the intermingled state (I) of two polymer chains exists.

(v) When $u = u_C$, $x = x_C$ and $\omega = \omega_C(u)$ the system evolves to a fixed point $(A^* = B^*, C^* = D^* = E^*) = (0.4406, 0.1941\dots)$. Linearization about this fixed point gives two eigenvalues greater than one, i.e., $\lambda_b = 2.7306\dots$ and $\lambda_c = 1.9476\dots$. These eigenvalues lead to the value of y equal to 0.6635.

In Fig. 4 a dashed line $\omega_C u_C$ represents the value of $\omega = \omega_C(u)$ at $x = x_C$ for which this fixed point is reached. This is a tricritical line and separate the phases indicated by I and E . The E phase represents the state in which the two chains are zipped together.

The line $u_C \omega_C$ meets the plane $\omega = 0$ at point (u_C, x_C) which itself is a tricritical point. Since at this point $\omega = 0$, the configurations C and D do not survive at any length scale whereas for any other point on the line $u_C \omega_C$ these configurations exist at all length scales.

The point T_1 on this line corresponds the value of $\omega_C(u = 1.8147) = 1$. For $u < 1.8147$ and $\omega_C(u) > 1$, E phase does not exist and the line $u_C \omega_C$ separates the states I and C . Here C represents a phase in which two chains are entangled.

(vi) For $x < x_C$ and $u < u_C$ the system attains the following configurations.

(a) When $x'_C(u) < x < x_C$ and $u < u_C = 1.8577$ the fixed point to which the system evolves is $(A^* = B^*, C^* = D^*, E^*) = (0, 0, 0.7967)$. This fixed point as discussed in (iii) corresponds to a configuration in which a zipped long chain is formed. The individual chains for this case are non-critical.

(b) For $x < x'_C(u)$ and $w > w'_C(u)$ the fixed point attained by the system is $(A^* = B^*, C^*, D^*, E^*) = (0, 0.4406, 0, 0)$. This corresponds to the

configuration in which the two chains are entangled. We refer to this state of the chains as a C phase. In this case also the individual chains are non-critical.

(c) The fixed point $(A^* = B^*, C^*, D^* = E^*) = (0, 0.4406, 0, 0.7967)$ is reached for $x = x'_C(u)$ and $\omega = \omega'_C(u)$. The line $T_1 T_2$ in Fig. 4 represents the value of ω and x for given $u < u_C$ for which this fixed point is reached. Along the line $T_1 T_2$ the two uncoupled phases E and C coexist. The line $T_1 T_2$ appears to meet with the tricritical line $u_C w_C$ at point T_1 .

4. DISCUSSIONS

The problem with which we have been concerned here is to predict the effect of attraction between unlike monomers of two chemically different charge neutral homo polymers on their configurations. The system is shown to exist in a configuration of segregation in which they are separated from each other without any overlap, or in a state in which the two chains are either zipped together or entangled depending upon the attraction.

The system is represented by two interacting SAWs in such a way that these walks may cross each other at most once on a lattice point and a lattice bond may be occupied by at most a step of one or both walks, The crossing of chains on a lattice point is due to an attractive interaction between unlike monomers which characteristic length is shorter than that of the repulsion between like monomers. The model involves three parameters viz. fugacity x of a step of a walk, $\omega = \exp(E_C/T)$ and $t = \exp(E_t/T)$ where E_C and E_t , as noted above, are the contact energy and energy associated with an unoccupied lattice bond adjacent to a doubly visited site. All results given above are for $t = 0.5$. The value of the contact exponent y tabulated above for the Sierpinski gasket type fractals shows that it decreases monotonically as the parameter b increases.

It is also shown that for large b , $y = 2 - \nu(b) d_F$ and therefore, equals to α , i.e. the number of contacts formed between chains scale in the same way as the number of closed loops formed by a SAW in its asymptotic limit. This result was found by using a phenomenological approach based on the finite-size-scaling theory in which one is concerned with the change in the renormalization equation as scale factor is changed. This differs from the usual case, where the renormalization equations are unchanged as the lengths are rescaled but the coupling constants are changed so that correlation functions of a Hamiltonian at a length scale L , are related to those the transformed Hamiltonian at length scale L/b . In the treatment given above, one determines how the recursion equation for the b -fractal are related to

those of (say) $2b$ -fractal. This analysis clearly indicates that $y = \alpha$ is satisfied only for a lattice for which the resealing parameter b approaches infinity.

The model which we solved for a 4-simplex lattice has an additional parameter, viz., $u = \exp(E_n/T)$ where E_n is the attraction energy between a pair of unlike monomers occupying nearest neighbor lattice sites. It is shown that the two attraction parameters ω and u compete with each other. While ω favors formation of entangled configuration of two chains, u , on the other hand, favors zipping of the chains. The phase diagram plotted in Fig. 4 shows the region of existence of these phases.

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